

1 Fundamentals of distributions

2 Discrete distributions

2.1 Uniform distribution

2.2 Binomial distribution

The binomial distribution gives the number of 1's when a Bernoulli trial that attains 1 with probability p is repeated n times.

PMF:

$$f(x) = f(x|n, p) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \dots, n$$

By the Binomial theorem:

$$\begin{aligned} 1 &= (p + (1-p))^n \\ &= \sum_{x=0}^n \binom{n}{x} p^x (1-p)^{n-x} \end{aligned}$$

Expected value, variance, and MGF

$$\begin{aligned} EX &= np \\ \text{Var} X &= np(1-p) \\ M(t) &= (pe^t + 1 - p) \end{aligned}$$

2.3 Negative binomial distribution

The negative binomial distribution gives the number of 1's before a sequence of Bernoulli trials each of which attains 1 with probability p generates r number of 0's.

PMF:

$$f(x) = f(x|r, p) = \binom{r+x-1}{x} p^r (1-p)^x, \quad x = 0, 1, 2, \dots$$

Expected value, variance, and MGF

$$\begin{aligned} EX &= \frac{r(1-p)}{p} \\ \text{Var} X &= \frac{r(1-p)}{p^2} \\ M(t) &= \left(\frac{p}{1 - (1-p)e^t} \right)^r, \quad t < -\ln(1-p) \end{aligned}$$

2.4 Poisson distribution

PMF:

$$f(x) = f(x|\theta) = e^{-\theta} \frac{\theta^x}{x!}, \quad x = 0, 1, 2, \dots$$

Expected value, variance, and MGF

$$M(t) = \sum_{x=0}^{\infty} e^{xt} e^{-\theta} \frac{\theta^x}{x!} = \exp(\theta(e^t - 1))$$

We obtain the expected value and variance by differentiating the MGF

$$\begin{aligned} EX &= M'(t) = \theta \\ \text{Var}X &= M''(t) = \theta \end{aligned}$$

Properties: If $X \sim \text{Poi}(\theta_1)$ and $Y \sim \text{Poi}(\theta_2)$, and $X \perp Y$, then

$$M_{X+Y}(t) = M_X(t)M_Y(t) = \exp((\theta_1 + \theta_2)(e^t - 1))$$

and so

$$X + Y \sim \text{Poi}(\theta_1 + \theta_2)$$

3 Continuous distributions

3.1 Normal distribution

3.2 Exponential distribution

3.3 Beta distribution

3.4 Gamma distribution

3.5 Weibull distribution

3.6 Cauchy distribution

3.7 Log-normal distribution

3.8 Rayleigh distribution

3.9 Tukey-Lambda distribution

3.10 Generalised extreme value distribution

3.11 Logistic distribution

3.12 Levy distribution

3.13 Laplace distribution

3.14 Triangular distribution

3.15 F distribution

3.16 Student's t distribution

3.17 Chi-squared distribution

3.18 Erlang distribution