1 Inequalities

1.1 Triangle inequality

In an n-dimensional vector space, the shortest path from A to B is a straight line:

$$|\mathbf{x}+\mathbf{y}| \leq ||\mathbf{x}|| + ||\mathbf{y}||$$

1.2 Hölder inequality

Let $p, q > 1$ be such quantities that

$$\frac{1}{p} + \frac{1}{q} = 1$$

Then

$$\mathbb{E}\,|XY| \leq [\mathbb{E}\,|X|^p]^{\frac{1}{p}} [\mathbb{E}\,|Y|^q]^{\frac{1}{q}}$$

1.3 Cauchy-Schwarz

The Cauchy-Schwarz inequality is a special case of the Hölder inequality where $p = q = 2$:

$$|\,\mathbb{E}(XY)| \leq \mathbb{E}\,|XY| \leq \sqrt{\mathbb{E}\,X^2}\sqrt{\mathbb{E}\,Y^2}$$

Alternatively,

$$|\langle \mathbf{x} \cdot \mathbf{y} \rangle| \leq ||\mathbf{x}|| \cdot ||\mathbf{y}||$$

$$\mathbb{E}[|f \cdot h|] \leq \sqrt{\mathbb{E}[f^2]\,\mathbb{E}[h^2]}$$

1.4 Markov inequality

$$P(X \geq a) \leq \frac{\mathbb{E}\,X}{a} \quad , \quad \forall a > 0$$

1.5 Chebyshov inequality

Applying the Markov inequality to the random variable $Y = (X - \mu)^2$ we have the Chebyshov inequality:

$$P(|X - \mu| \geq t) \leq \frac{\sigma^2}{t^2} \quad , \quad \forall t > 0$$

1.6 Jensen's inequality

Let $I \subset R$ be an open interval and $g : I \to \mathbb{R}$ a convex function. If X is a random variable with values in I (with probability 1), and both $\mathbb{E}\,X$ and $\mathbb{E}\,g(X)$ exist, then

$$g(\mathbb{E}\,X) \leq \mathbb{E}\,g(X)$$