

1 Properties of functions

1.1 Convexity

Let $\Delta \subset R$ be an interval. Function $f : \Delta \rightarrow R$ is **convex**, if

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y) \quad \forall x, y \in \Delta \quad \text{and} \quad \lambda \in [0, 1]$$

Function $f : \Delta \rightarrow R$ is **concave**, if $-f$ is convex.

1.2 Linearity

A **linear map** is a function that satisfies the following two properties.

Additivity:

$$f(x + y) = f(x) + f(y)$$

Homogeneity of degree one:

$$f(\alpha x) = \alpha f(x) \quad , \quad \forall \alpha$$

1.3 Superlinearity / superadditivity

$$f(x + y) \geq f(x) + f(y)$$

1.4 Sublinearity / subadditivity

$$f(x + y) \leq f(x) + f(y)$$

1.5 Homogeneity

A function where

$$f(\alpha x_1, \dots, \alpha x_n) = \alpha^k f(x_1, \dots, x_n)$$

is called a **homogeneous function of degree k**.