Tricks

"A trick used twice is a method."

Log/exponential transform

- $(1+i)^x = e^{x\log(1+i)} = \exp[x\log(1+i)]$
- $x^2 = e^{\log x^2} = e^{2\log x} = \log(e^{x^2}) = x^2$
- $\theta = e^{\log\theta} = e^{\log\theta} = \log(e^\theta) = \theta$

Scaling

$$\begin{aligned} f(x) = 2x &\quad\Longrightarrow\quad \frac{1}{2}f(x) = x \\ \int_a^b g(x) = \sqrt{2\pi} &\quad\Longrightarrow\quad \frac{1}{\sqrt{2\pi}}\int_a^b g(x) = 1 \\ \sum_{i=1}^{T} A(x) = \log\psi &\quad\Longrightarrow\quad \frac{1}{\log\psi}\sum_{i=1}^{T} A(x) = 1 \end{aligned}$$

Derivative substitution

$$\lim_{q\to 0}\frac{\sqrt{1+q}-1}{q} = D_q\sqrt{q}\bigg|_{q=1} = \frac{1}{2\sqrt{q}}\bigg|_{q=1} = \frac{1}{2}$$

Trig substitution

Inverse substitution

Let $f(x) = exp(x) = e^x$ and $f^{-1}(x) = \ln x$. Then

$$D_x \ln x = D_x f^{-1}[f(\ln x)] = \frac{1}{f'(\ln x)} = \frac{1}{e^{\ln x}} = \frac{1}{x}$$

Taylor substitution

$$e^x = \sum_{k=0}^{\infty}\frac{x^k}{k!}$$

Multiply by one

$$x = x \cdot \frac{n}{n} = x \cdot \frac{1}{n} \cdot n = \frac{x}{n} \cdot n = x$$

Add-and-subtract

$$x = x + y - y = x$$

Regrouping

$$\begin{aligned} 1 - 1 + 1 - 1 + 1 - 1 + ... &\neq (1-1) + (1-1) + (1-1) + ... \\ &= 0 + 0 + 0 + ... \\ &= 0 \end{aligned}$$

$$\begin{aligned} S &= 1 + 2 + 3 + ... + n = n + (n-1) + (n-2) + ... + 3 + 2 + 1 \\ 2S &= 1 + n + 2 + (n-1) + ... + 2 + (n-1) + 1 + n \\ 2S &= n(n+1) \end{aligned}$$

Add-and-subtract and regroup

$$(x - z) = (x - y + y - z) = (x - y) + (y - z) = (x - z)$$

System of equations from a polynomial

$$\begin{aligned} Ax^2 + Bx^2 + Cx + Bx + A + C &= 4x^2 + 5 \\ \iff (A+B)x^2 + (B+C)x + (A+C) &= 4x^2 + 0x + 5 \end{aligned}$$

we get a system of equations

$$\begin{aligned} A + B &= 4 \\ B + C &= 0 \\ A + C &= 5 \end{aligned}$$

and find that

$$\begin{aligned} A &= 9/2 \\ B &= -1/2 \\ C &= 1/2 \end{aligned}$$

Partial fractions

$$\begin{aligned} \frac{1}{x(1+x)} &= \frac{A}{x} + \frac{B}{1+x} \\ &\implies A(1+x) + Bx = 1 \\ &\implies A + Ax + Bx = 1 \\ &\implies (A+B)x + A = (0)x + 1 \end{aligned}$$

We now get a system of equations

$$\begin{aligned} A &= 1 \\ Ax + Bx &= 0 \end{aligned}$$

And find that

$$\begin{aligned} A &= 1 \\ B &= -1 \end{aligned}$$